J. Fluid Mech. (1972), *vol.* 56, *part* 3, *pp.* 607–608
Printed in Great Britain

REVIEW

The Thermodynamics of Simple Materials with Fading Memory.

By W. A. DAY. Springer-Verlag, 1972. 134 pp. DM44 or $13.70.

The initial aims of the thermodynamics of continuous media are first to discover the appropriate form of the irreversibility axiom, or second law, and then to determine the restrictions imposed by that law on the constitutive equations defining extensive classes of ideal materials. Much of the recent work in this area is based upon the premise that the Clausius–Duhem inequality, or a variant of it, applying to an arbitrary part of a material body or to the whole body, provides a universally valid expression of the concept of irreversibility. This approach has major attractions. The task of selecting from the class of functions or functionals admitted by a constitutive postulate those which are compatible with the inequality as well as the balance laws of continuum mechanics becomes a straightforward problem which can be solved economically, even elegantly, and the results obtained for elastic materials and for viscous fluids are precisely those which have long been accepted on the basis of different arguments. Further, a study of behaviour near equilibrium yields theorems on the dissipation of energy and the extremal properties of free energy and entropy which accord with corresponding ideas in classical thermostatics. But the adoption of an entropy production inequality of the Clausius–Duhem type can be questioned, on two grounds at least. First, it requires the acceptance of entropy as one of the primitive concepts of continuum physics, not derivable from ideas considered more fundamental; and second, this sort of irreversibility postulate may be too strong to the extent that an alternative form might still deliver the restrictions on elastic and viscous behaviour referred to above while permitting in other kinds of materials, notably those exhibiting memory effects, a diversity of response greater than that allowed by an entropy production inequality.

In Chapters 2 to 4 of the monograph under review Dr Day has developed an approach to the thermodynamics of materials with fading memory in which the second law is an inequality, not involving entropy, applying to cyclic processes starting from equilibrium. The notion of fading memory is embodied in a definition which is intuitively motivated and sufficiently weak to bring within the ambit of the theory a very wide variety of material response, including as extreme cases elasticity and viscosity. The core of the book, consisting of Chapters 3 and 4, describes how the entropy functional can be constructed, first for equilibrium processes and then for processes starting from equilibrium and proceeding in an arbitrary fashion, and how the entropy can be found explicitly in certain cases; in particular for elastic materials, for differential-type materials (of which the Newtonian viscous fluid is one), and for a rather special class of viscoelastic materials. These chapters are based upon a series of papers published by the author since 1968, but in the systematic presentation given here some significant revisions and extensions of the earlier work are apparent.

There follows in Chapter 5 an account of the thermodynamics of materials

with fading memory based upon the Clausius–Duhem inequality. In scope and in spirit this chapter is modelled on the original work of Coleman, published in 1964, but full advantage has been taken of subsequent clarifications. Coleman's theory calls for an analytically motivated characterization of fading memory stronger, and less appealing, than the form introduced by the author in Chapter 2, and the class of materials embraced by the theory is consequently smaller, materials of the differential type no longer being included. There emerges from Chapter 5 a clear appreciation of the price to be paid, in respect of materials with genuine memory, when a requirement of non-negative entropy production is replaced by a less restrictive irreversibility postulate of the kind proposed by the author. The construction of the entropy for this class of materials is a formidable undertaking and only in special cases has it been found possible as yet to calculate the entropy explicitly. In compensation, a more extensive range of materials can be brought within the compass of a unified thermomechanical theory, and the detailed analysis, although mathematically quite demanding, is more closely related to operational procedures of the kind encountered in physically based treatments of classical thermostatics.

The opening chapter of the book provides a succinct survey of the results in vector and tensor analysis and the basic topics in continuum mechanics needed in the later developments. The final chapter, again largely based upon recent research by the author, explores thermodynamic restrictions on the relaxation function specifying the isothermal response of a viscoelastic material to small deformations.

Dr Day has written a readable and stimulating text marred only by a sprinkling of minor misprints of a kind likely to mislead the careful reader. He has performed a valuable service by making available, in a well organized and clearly written exposition, recent work on the thermodynamics of continuous media which promises to exert a strong influence on future developments in this important field of research.

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